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ACOUSTIC DISPERSION IN RAREFIED GASES

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The problem of acoustic dispersion in rarefied gases is solved on the basis of the hydrodynamical equations of Predvoditelev. The theoretical equation is compared with the experiments of Greenspan for five monatomic gases. Theory and experiment are compared up to a Knudsen number of order unity.

1. On the Nonideal Continuity Parameter

In 1948 A. S. Predvoditelev described a technique for improving the Navier–Stokes equations in application to problems in which the hydrodynamic velocity gradient is related to the path traversed by the molecules between collisions. This technique is based essentially on the Maxwell transport equation and a more precise hypothesis regarding the relationship between the hydrodynamic flow velocity and the transport velocities of two colliding molecules. The indicated relationship must be determined in transforming from the Maxwell transport equation to the continuum equations.

If the most general assumptions are advanced with regard to the transport velocities of two colliding molecules, the equations for the hydrodynamic stresses have the form [2]

$$\left.\begin{aligned} \rho\overline{\xi_i^2} &= p + \frac{A_1}{3A_2}\rho\left(v_{2i}^2 - v_{1i}^2\right) - 2\mu\left(\frac{\partial v_i}{\partial x_i} - \frac{\gamma-1}{2}\operatorname{div}\mathbf{V}\right) \\ \rho\overline{\xi_i\xi_j} &= -\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \left(v_{2i}v_{2j} - v_{1i}v_{1j}\right);\quad i = 1,\ 2,\ 3, \\ & j = 1,\ 2,\ 3. \end{aligned}\right\} \tag{1.1}$$

When the transport velocities of the two colliding molecules are equal, i.e., when $v_{2i} = v_{1i} = v_i$, Eqs. (1.1) go over to the expressions derived in the theory of the Navier–Stokes equations. Equations (1.1) can be used, however, to obtain the more complete Predvoditelev equations in the form [2]

$$\rho\frac{dv_i}{dt} + \sum_j \frac{A_1}{3A_2}\cdot\frac{\partial}{\partial x_j}\left[\rho\left(v_{2i}v_{2j} - v_{1i}v_{1j}\right)\right] = -\frac{\partial p}{\partial x_i} + \mu\left[\nabla^2 v_i + (2-\gamma)\frac{\partial}{\partial x_i}\operatorname{div}\mathbf{V}\right]. \tag{1.2}$$

It is important to note that in the Maxwell calculations $(2-\gamma) = 1/3$, but this expression is valid only for a monatomic gas. Consequently, this restriction of Maxwell is tacitly implicit in the adiabatic equation for a monatomic gas. The latter fact was also first noted by Predvoditelev [1].

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We transform the terms of Eq. (1.2) into dimensionless form. Equations (1.1) are used to select the following scale for the transport velocities of the colliding molecules:

$$[v_{2i}] = [v_{1i}] = \sqrt{\frac{\mu U}{\rho_0 L}}. \tag{1.3}$$

Here ρ_0, U, and L are the density, velocity, and length scales, respectively. Adopting the customary scales for all other quantities, we rewrite the system (1.2) in the dimensionless form

$$\rho\frac{dv_i}{dt} + \frac{A_1 K}{3A_2}\sum_{j=1}^{3}\frac{\partial}{\partial x_j}[\rho(v_{2i}v_{2j} - v_{1i}v_{1j})] = -\frac{\partial p}{\partial x_i} + \frac{1}{\mathrm{Re}}\left[\nabla^2 v_i + (2-\gamma)\frac{\partial}{\partial x_i}\operatorname{div}\mathbf{V}\right], \tag{1.4}$$

where

$$K = \frac{\mu U}{p_0 L} = \frac{\mu}{\rho_0 L U} = \frac{1}{\mathrm{Re}}. \tag{1.5}$$

To close the system (1.4) we invoke the hypothesis

$$\left.\begin{aligned} v_{1i} &= v_i + A\sum_{j=1}^{3}(x_j - x_{0j})\frac{\partial v_i}{\partial x_j}, \\ v_{2i} &= v_i - A\sum_{j=1}^{3}(x_j - x_{0j})\frac{\partial v_i}{\partial x_j}, \quad i = 1,\ 2,\ 3. \end{aligned}\right\} \tag{1.6}$$

This brings us to the original equations of Predvoditelev [1]:

$$\rho\frac{\partial \mathbf{V}}{\partial t} + \rho\left[\frac{1}{2}(1-\beta)\operatorname{grad}V^2 - \beta\mathbf{V}\operatorname{div}\mathbf{V} + (1-\beta)\operatorname{rot}\mathbf{V}\times\mathbf{V}\right] = \operatorname{grad}p + \frac{1}{\mathrm{Re}}\left[\nabla^2\mathbf{V} + (2-\gamma)\operatorname{grad}\operatorname{div}\mathbf{V}\right]. \tag{1.7}$$

Predvoditelev called the quantity $\beta = A_1AK/3A_2$ the nonideal continuity parameter. Knowing that A_1 and A_2 are numbers, we infer that the parameter β contains only one experimental constant A. It is evident from (1.6) that it determines the magnitude and direction of the gradient and can therefore have any sign.

Inasmuch as acoustic motions do not have a characteristic velocity, the Reynolds number is given as

$$\mathrm{Re} = r = \frac{g_0^2\rho}{\gamma\omega\mu}. \tag{1.8}$$

Now the parameter $\beta = \beta_0/r$.

2. Acoustic Dispersion

The acoustic dispersion equations are readily obtained by using the Hugoniot–Hadamard compatibility conditions. These conditions are known to characterize the formation and propagation of a wave front. They make it possible to find the velocity of the discontinuity surfaces in space solely by determining those surfaces, without integration. Predvoditelev [3] has elaborated the method of Hadamard in application to acoustical problems, so we need not delve at length into the details of the mathematical aspects of the problem.

To solve the stated problem we use the one-dimensional equations (1.7) in the dimensional form

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + (3-\gamma)\mu\frac{\partial^2 u}{\partial x^2} + 2\beta\rho u\frac{\partial u}{\partial x}, \tag{2.1}$$

the well-known heat-transfer equation

$$\rho c_v\frac{\partial T}{\partial t} + p\frac{\partial u}{\partial x} = k\frac{\partial^2 T}{\partial x^2}, \tag{2.2}$$

and the equation of continuity

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial x} = 0. \tag{2.3}$$

Here, as is the custom in linear acoustics, we neglect the influence of acoustic streaming.

To Eq. (2.1) we apply the operation of translation across the fronts by means of the identity and kinematic conditions of Hadamard. We have as a result

$$\frac{\lambda_{1p}}{\lambda_{1u}} = \rho g + 2\beta\rho u + (3-\gamma)\mu \frac{\lambda_{2u}}{\lambda_{1u}}. \tag{2.4}$$

But the equation of continuity implies

$$\lambda_{1\rho} g = \rho\lambda_{1u}. \tag{2.5}$$

Furthermore, it can be shown that the following equation holds for periodic motions:

$$\frac{\lambda_{2u}}{\lambda_{1u}} = -\frac{i\omega}{g}.$$

In light of these remarks we now rewrite Eq. (2.4) in the form

$$\frac{\lambda_{1p}}{\lambda_{1\rho}} = g^2 + 2\tilde{\alpha}\beta g_0^2 - (3-\gamma)\frac{\mu}{\rho}\omega i. \tag{2.6}$$

Here the velocity u is eliminated on the basis of the hypothesis

$$u = \frac{\tilde{\alpha} g_0^2}{g}. \tag{2.7}$$

To Eq. (2.2) we apply the operation of translation across the acoustic wave fronts. We then have

$$-\rho c_v g\lambda_{1T} + p\lambda_{1u} = k\lambda_{2T}.$$

Alternatively, recognizing that $g_0^2 = \gamma(p/\rho)$, we rewrite the latter equation in the form

$$\frac{g_0^2}{\gamma} = c_v \frac{\lambda_{1T}}{\lambda_{1u}} + \frac{k}{\rho} \cdot \frac{\lambda_{2T}}{\lambda_{1u}}. \tag{2.8}$$

We use the equation of state for an ideal gas:

$$p = \rho RT,$$

which can also be written

$$\lg p = \lg R + \lg T + \lg \rho.$$

We take the time derivative:

$$\frac{1}{p} \cdot \frac{dp}{dt} = \frac{1}{T} \cdot \frac{\partial T}{\partial t} + \frac{1}{\rho} \cdot \frac{\partial \rho}{\partial t}.$$

Next, having applied the operation of translation across the acoustic wave fronts, we represent the state equation in the form

$$\frac{\lambda_{1p}}{\lambda_{1\rho}} = \frac{p}{\rho} + \frac{p}{T} \cdot \frac{\lambda_{1T}}{\lambda_{1\rho}}. \tag{2.9}$$

By means of relation (2.5) and Eq. (2.9) we transform Eq. (2.8) as follows:

$$\frac{\lambda_{1p}}{\lambda_{1\rho}} = \frac{g_0^2}{\gamma} + \frac{\gamma-1}{\gamma} \cdot \frac{g_0^2}{\left[1 + \frac{a}{g} \cdot \frac{\lambda_{2T}}{\lambda_{1T}}\right]}. \tag{2.10}$$

We assume that the temperature oscillation in the acoustic wave takes place at the frequency of the velocity oscillations. From this assumption we deduce the equation

$$\frac{\lambda_{2T}}{\lambda_{1T}} = -\frac{i\omega}{g}.$$

Now, consolidating (2.6) and (2.10), we have

$$\frac{g_0^2}{\gamma} + \frac{\gamma-1}{\gamma} \cdot \frac{g_0^2}{\left(1 - \frac{a}{g^2}\omega i\right)} = g^2 + 2\tilde{\alpha}\beta g_0^2 - (3-\gamma)\frac{\mu}{\rho}\omega i.$$

Separating the real and imaginary parts in the latter equation, we represent the complex velocity of sound as follows:

$$g = g_1 + ig_2.$$

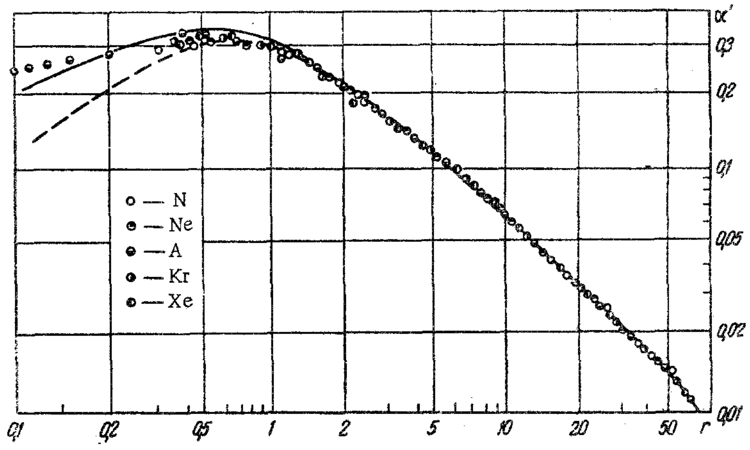

Fig. 1. Results of measurements and theory for five monatomic gases. Solid curve) proposed theory; dashed curve) Barnett approximation.

We recall here that the quantity g_2 is small in comparison with g_1, because the sound absorption coefficient is always small. We also use the well-known Eiken relation

$$\frac{k}{\mu c_v} = \frac{1}{4}(9\gamma - 5) = \gamma f,$$

whereupon we readily deduce the equations

$$\left.\begin{aligned} g_1^2 &= g_0^2\left[\left(1 - \frac{2\alpha_0}{r}\right) + \frac{(3-\gamma)}{\gamma}\cdot\frac{f}{r^2}b_1\right], \\ 2g_1g_2 &= \frac{g_0^2}{r}\left[\left(\frac{3-\gamma}{\gamma} + f\right) - \frac{f}{\gamma}b_1 + \frac{2\alpha_0 f}{r}b_1\right]. \end{aligned}\right\} \tag{2.11}$$

The following additional notation is introduced here:

$$b_1 = \frac{g_0^2}{g_1^2}\ ;\quad \alpha_0 = \tilde{\alpha}\beta_0.$$

If we neglect g_2^2 in comparison with g_1^2, we can show at once that the absorption coefficient is

$$\alpha = \frac{\omega g_2}{g_1^2}\,. \tag{2.12}$$

Normally, the quantity $\alpha' = \alpha g_0/\omega$ is involved in the processing of the experimental data [4]. Now by well-known rules we readily obtain from (2.11)

$$\alpha' = \frac{\sqrt{2r}(Ar - B\alpha_0)}{(r-2\alpha_0)^2\sqrt{(r-2\alpha_0)(1+\sqrt{1+m})}}\,, \tag{2.13}$$

where

$$\left.\begin{aligned} m &= \frac{(3-\gamma)(9\gamma-5)}{\gamma^2(r-2\alpha_0)^2}\,, \\ A &= \frac{(7\gamma+5\gamma^2)(1+\sqrt{1+m}) - 2(9\gamma-5)}{4\gamma^2(2+2\sqrt{1+m}+m)}\,, \\ B &= \frac{(7+5\gamma)(1+\sqrt{1+m}) + 2(5-9\gamma)}{2\gamma(2+2\sqrt{1+m}+m)}\,. \end{aligned}\right. \tag{2.14}$$

If in (2.13) we set $m = 0$, $\alpha_0 = -0.2875$, and $\gamma = 5/3$, as is consistent with a monatomic gas, Eq. (2.13) then goes over to the Predvoditelev expression [3]

$$\alpha' = \frac{\sqrt{r}(0.7r + 0.23)}{(r+0.575)^2\sqrt{r+0.575}}\,. \tag{2.15}$$

However, it is obvious from (2.15) that m can be approximately equal to zero only for large values of r, i.e., for dense gases. In the case of monatomic gases, therefore, Eq. (2.13) refines not only (2.15), but also the experimental constant α_0. To verify Eq. (2.13) we refer to the experiments conducted by Greenspan on five monatomic gases [4], along with the results of Meyer and Sessler [5] (see Fig. 1). The experiments show that all five gases behave identically. Equation (2.13) is applied to the experimental results for $\alpha_0 = -0.12$. It is evident from Fig. 1 that agreement between theory and experiment is obtained up to $r = 0.15$, which corresponds to a Knudsen number of order unity [5].

The Predvoditelev equations can therefore be used in describing rarefied gas flows up to Knudsen numbers close to unity.

NOTATION

V, hydrodynamic velocity vector; μ, viscosity; ρ, density; γ, specific heat ratio; g_0, Laplace value of the velocity of sound; ω, cyclic frequency; c_V, specific heat at constant volume; g, phase velocity of sound; λ_{1u}, λ_{1o}, $\lambda_{1\rho}$, λ_{1T}, parameters of first-order discontinuity; λ_{2u}, λ_{2T}, parameters of second-order discontinuity; k, thermal conductivity; R, universal gas constant.

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FLOW AND HEAT TRANSFER IN A JET NEAR THE STAGNATION POINT OF A CONCAVE BODY

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Results are presented of calculations of flow and heat transfer near the stagnation point of a concave body in a two-dimensional subsonic jet, using a flow establishment method.

The interaction of a jet flow with blunt bodies is usually taken to mean the flow near the stagnation point, outside the region influenced by the body shape. We consider the problem of specifying such a flow near the surface of a concave body of constant curvature, located in a subsonic jet. The flow is assumed to be two-dimensional, and the fluid is assumed to be incompressible and viscous near the body surface. We restrict the analysis to a small region near the stagnation point, and represent the flow of the jet far from the surface as being approximately the flow from an ideal source.

In the body-fixed coordinate system $(\bar{\xi}, \bar{\zeta})$, (Fig. 1), where the $\bar{\xi}$ axis is tangent to the body surface, and the $\bar{\zeta}$ axis is normal to it, we select the section $\bar{\zeta} = \bar{\zeta}_\infty$, where the source flow velocity is known and equal to $\bar{V}_\infty$. We consider that the section $\bar{\zeta}_\infty$ is at a considerable distance from the obstacle, so that the effect of the obstacle on the source flow is negligibly small here. The flow is symmetric relative to the obstacle center $\bar{\xi} = \bar{\zeta} = 0$, and the external flow is irrotational; the effect of viscosity is localized in a thin boundary layer near the obstacle surface. The problem is solved in two stages. In the first stage we seek a solution in the region where the source flow and the obstacle interact $(0 \le \bar{\zeta} \le \bar{\zeta}_\infty)$, and we formulate boundary conditions for the viscous flow and heat transfer in the obstacle boundary layer. In the second stage we consider the establishment of a boundary layer on the obstacle, and determine the friction τ_W and the heat flux q_W to the surface. The problem is solved by a flow establishment method, applied to the unsteady boundary-layer equa-

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